IS3320
Advanced Quantitative Analysis

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Course Map

Week	Lecture	Tutorial 5%	Lab 10%	Exam 40%	Assignment 5%
1	Academic Advising				
2-5	√	√	√		√
6	√	√		√	
7-11	√	√	√		√
12	√	√		√	√
13-15	√	√	√		√
17	Final exam 40%				

1 Topics to be Covered		
List of Topics	No of Weeks	Contact hours
Transportation Problem	3	9
Assignment Models	2	6
Linear and Nonlinear Programming	3	9
Integer Programming and Goal Programming	3	9
Waiting Lines and Queuing Theory Models	2	6
Simulation Modeling	2	6

Course No: IS3320	الرقم والرمز: ٣٣٢٠نال
Course: Advanced Quantitative Analysis	إسم المقرر: التحليل الكمي المتقدم
Credits: 3 (3 + 1 + 0)	الساعات: ٣ (٣ + ١ + ٠)
Pre-requisite: IS3310	متطلب سابق: ٣٣١٠نال

نماذج إتّخاذ القرار المتقدّمة في حلّ مشاكل حالات الأعمال؛ نماذج النقل، برمجة الأعداد الصحيحة، برمجة الهدف، تطبيقات المحاكاة، عملية Markov في حصص السوق والبرمجة الديناميكية؛ دراسة واقعية.

Advanced decision making models in solving business case problems; transportation models, integer programming, goal programming, simulation applications, Markov process in market shares and dynamic programming; case study.

Text Book

"Quantitative Analysis for Management" by ,Barry Render, Ralph M. Stair, and Michael E. Hanna

Prentice Hall Latest Edition

Chapter 10

Transportation and Assignment Models

To accompany
Quantitative Analysis for Management, Tenth Edition,
by Render, Stair, and Hanna
Power Point slides created by Jeff Heyl

Learning Objectives

After completing this chapter, students will be able to:

1. Structure special LP problems using the transportation and assignment models
2. Use the northwest corner, VAM, MODI, and stepping-stone methods
3. Solve facility location and other application problems with transportation models
4. Solve assignment problems with the Hungarian (matrix reduction) method

Chapter Outline

Chapter Outline

10.8 Degeneracy in Transportation Problems
10.9 More Than One Optimal Solution
10.10 Maximization Transportation Problems
10.11 Unacceptable or Prohibited Routes
10.12 Facility Location Analysis
10.13 Assignment Model Approach
10.14 Unbalanced Assignment Problems
10.15 Maximization Assignment Problems

Introduction

- In this chapter we will explore two special linear programming models
  - The transportation model
  - The assignment model
- Because of their structure, they can be solved more efficiently than the simplex method
- These problems are members of a category of LP techniques called ***network flow problems***

Introduction

- Transportation model
  - The *transportation problem* deals with the distribution of goods from several points of supply (*sources*) to a number of points of demand (*destinations*)
  - Usually we are given the capacity of goods at each source and the requirements at each destination
  - Typically the objective is to minimize total transportation and production costs

Introduction

- Example of a transportation problem in a network format

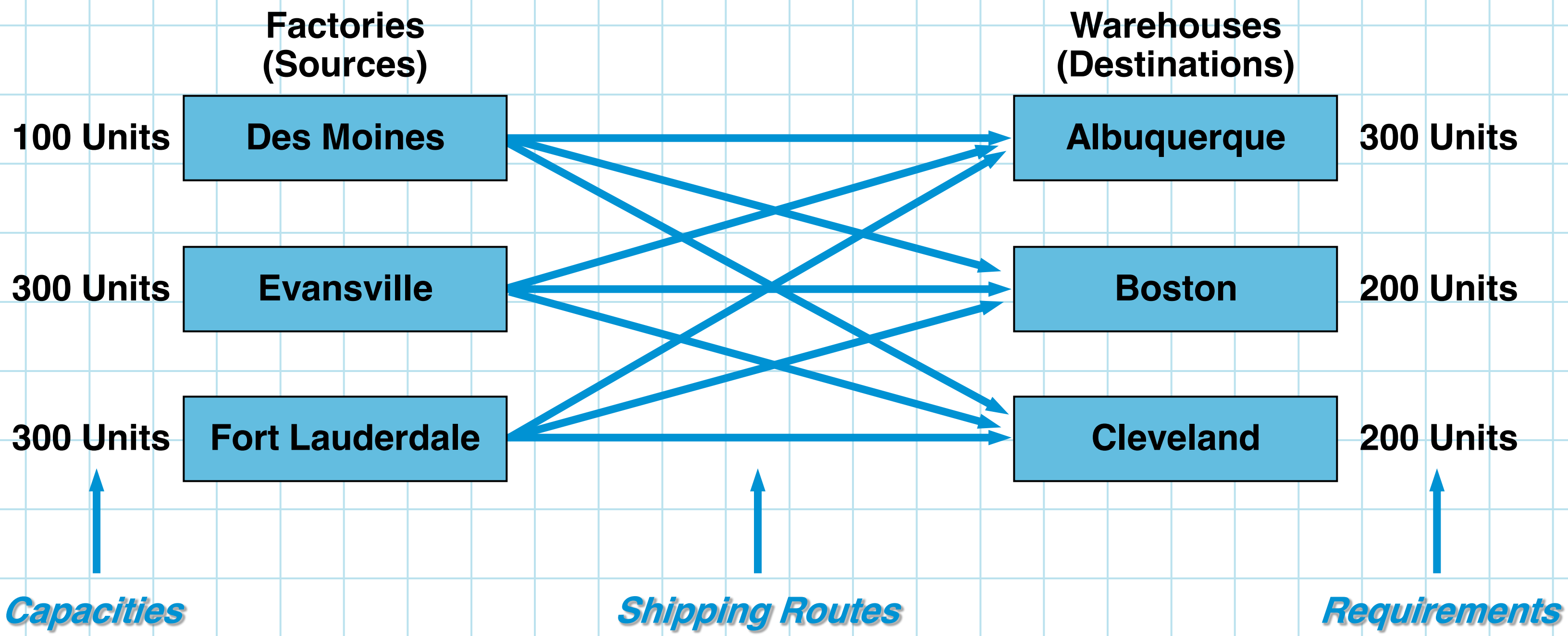


Figure 10.1

Introduction

- Assignment model
  - The *assignment problem* refers to the class of LP problems that involve determining the most efficient assignment of resources to tasks
  - The objective is most often to minimize total costs or total time to perform the tasks at hand
  - One important characteristic of assignment problems is that only one job or worker can be assigned to one machine or project

Introduction

- Special-purpose algorithms
  - Although standard LP methods can be used to solve transportation and assignment problems, special-purpose algorithms have been developed that are more efficient
  - They still involve finding and initial solution and developing improved solutions until an optimal solution is reached
  - They are fairly simple in terms of computation

Introduction

- Streamlined versions of the simplex method are important for two reasons
  1. Their computation times are generally 100 times faster
  2. They require less computer memory (and hence can permit larger problems to be solved)
- Two common techniques for developing initial solutions are the northwest corner method and Vogel's approximation
- The initial solution is evaluated using either the stepping-stone method or the modified distribution (MODI) method
- We also introduce a solution procedure called the ***Hungarian method***, ***Flood's technique***, or the ***reduced matrix method***

Setting Up a Transportation Problem

- The Executive Furniture Corporation manufactures office desks at three locations: Des Moines, Evansville, and Fort Lauderdale
- The firm distributes the desks through regional warehouses located in Boston, Albuquerque, and Cleveland
- Estimates of the monthly production capacity of each factory and the desks needed at each warehouse are shown in Figure 10.1

Setting Up a Transportation Problem

- Production costs are the same at the three factories so the only relevant costs are shipping from each ***source*** to each ***destination***
- Costs are constant no matter the quantity shipped
- The transportation problem can be described as ***how to select the shipping routes to be used and the number of desks to be shipped on each route so as to minimize total transportation cost***
- Restrictions regarding factory capacities and warehouse requirements must be observed

Setting Up a Transportation Problem

- The first step is setting up the transportation table
- Its purpose is to summarize all the relevant data and keep track of algorithm computations

Transportation costs per desk for Executive Furniture

FROM \ TO	ALBUQUERQUE	BOSTON	CLEVELAND
DES MOINES	$5	$4	$3
EVANSVILLE	$8	$4	$3
FORT LAUDERDALE	$9	$7	$5

Table 10.1

Setting Up a Transportation Problem

- Geographical locations of Executive Furniture's factories and warehouses

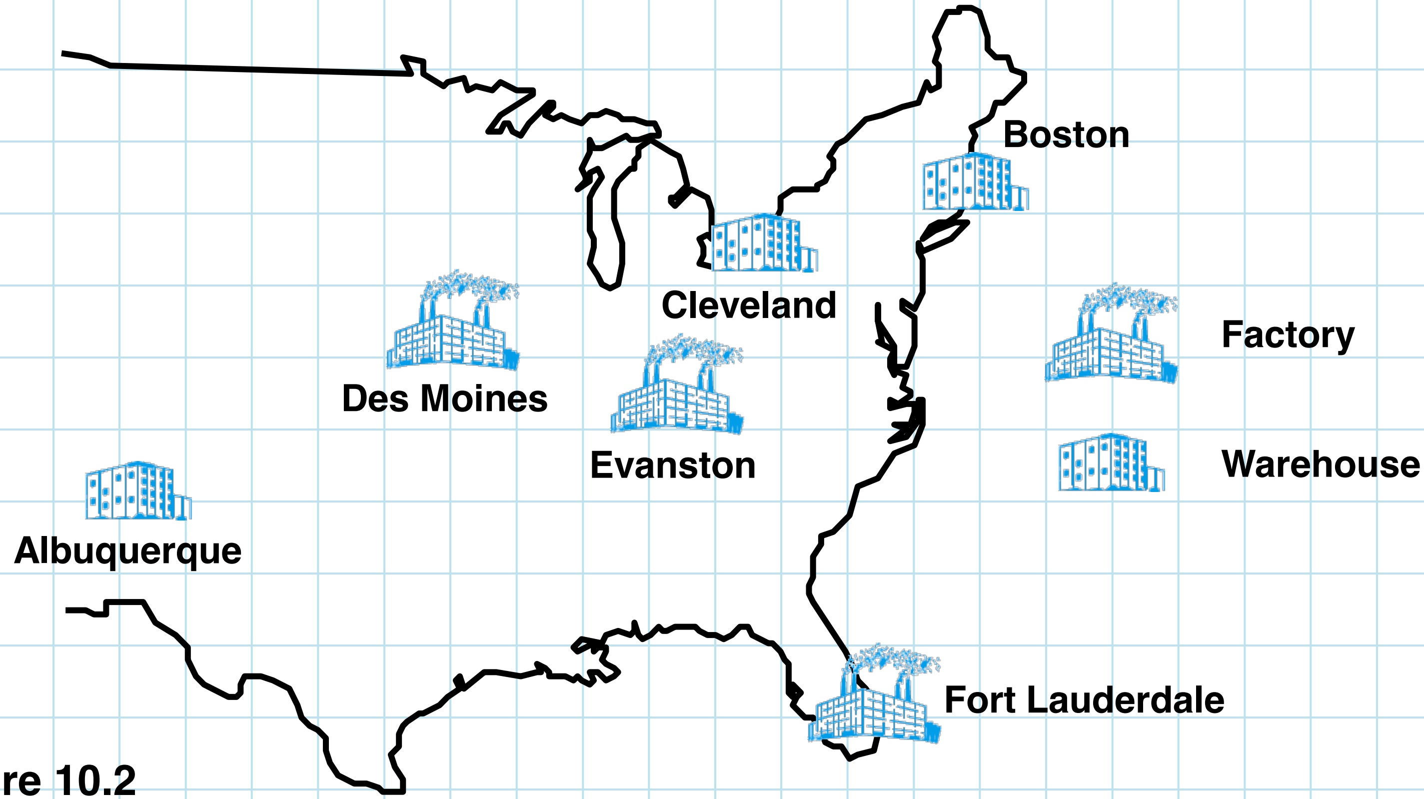


Figure 10.2

Setting Up a Transportation Problem

- **Transportation table for Executive Furniture**

Des Moines capacity constraint

FROM \ TO	WAREHOUSE AT ALBUQUERQUE	WAREHOUSE AT BOSTON	WAREHOUSE AT CLEVELAND	FACTORY CAPACITY
DES MOINES FACTORY	$5	$4	$3	100
EVANSVILLE FACTORY	$8	$4	$3	300
FORT LAUDERDALE FACTORY	$9	$7	$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 10.2

Cost of shipping 1 unit from Fort Lauderdale factory to Boston warehouse

Cleveland warehouse demand

Total supply and demand

Cell representing a source-to-destination (Evansville to Cleveland) shipping assignment that could be made

Setting Up a Transportation Problem

- In this table, total factory supply exactly equals total warehouse demand
- When equal demand and supply occur, a ***balanced problem*** is said to exist
- This is uncommon in the real world and we have techniques to deal with unbalanced problems

Developing an Initial Solution: Northwest Corner Rule

- Once we have arranged the data in a table, we must establish an initial feasible solution
- One systematic approach is known as the ***northwest corner rule***
- Start in the upper left-hand cell and allocate units to shipping routes as follows
  1. Exhaust the supply (factory capacity) of each row before moving down to the next row
  2. Exhaust the demand (warehouse) requirements of each column before moving to the right to the next column
  3. Check that all supply and demand requirements are met.
- In this problem it takes five steps to make the initial shipping assignments

Developing an Initial Solution: Northwest Corner Rule

1. Beginning in the upper left hand corner, we assign 100 units from Des Moines to Albuquerque. This exhaust the supply from Des Moines but leaves Albuquerque 200 desks short. We move to the second row in the same column.

TO / FROM	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	$5 100	$4	$3	100
EVANSVILLE (*E*)	$8	$4	$3	300
FORT LAUDERDALE (*F*)	$9	$7	$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Developing an Initial Solution: Northwest Corner Rule

2. Assign 200 units from Evansville to Albuquerque. This meets Albuquerque's demand. Evansville has 100 units remaining so we move to the right to the next column of the second row.

FROM \ TO	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	100 [$5]	[$4]	[$3]	100
EVANSVILLE (*E*)	200 [$8]	[$4]	[$3]	300
FORT LAUDERDALE (*F*)	[$9]	[$7]	[$5]	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Developing an Initial Solution: Northwest Corner Rule

3. Assign 100 units from Evansville to Boston. The Evansville supply has now been exhausted but Boston is still 100 units short. We move down vertically to the next row in the Boston column.

FROM \ TO	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	100 $5	$4	$3	100
EVANSVILLE (*E*)	200 $8	100 $4	$3	300
FORT LAUDERDALE (*F*)	$9	$7	$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Developing an Initial Solution: Northwest Corner Rule

4. Assign 100 units from Fort Lauderdale to Boston. This fulfills Boston's demand and Fort Lauderdale still has 200 units available.

FROM \ TO	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	100 $5	$4	$3	100
EVANSVILLE (*E*)	200 $8	100 $4	$3	300
FORT LAUDERDALE (*F*)	$9	100 $7	$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Developing an Initial Solution: Northwest Corner Rule

5. **Assign 200 units from Fort Lauderdale to Cleveland. This exhausts Fort Lauderdale's supply and Cleveland's demand. The initial shipment schedule is now complete.**

Table 10.3

FROM \ TO	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	100 | $5	| $4	| $3	100
EVANSVILLE (*E*)	200 | $8	100 | $4	| $3	300
FORT LAUDERDALE (*F*)	| $9	100 | $7	200 | $5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Developing an Initial Solution: Northwest Corner Rule

- We can easily compute the cost of this shipping assignment

ROUTE						
FROM	TO	UNITS SHIPPED	x	PER UNIT COST ($)	=	TOTAL COST ($)
D	*A*	100		5		500
E	*A*	200		8		1,600
E	*B*	100		4		400
F	*B*	100		7		700
F	*C*	200		5		1,000
						4,200

- This solution is feasible but we need to check to see if it is optimal

Stepping-Stone Method: Finding a Least Cost Solution

- The ***stepping-stone method*** is an iterative technique for moving from an initial feasible solution to an optimal feasible solution
- There are two distinct parts to the process
  - Testing the current solution to determine if improvement is possible
  - Making changes to the current solution to obtain an improved solution
- This process continues until the optimal solution is reached

Stepping-Stone Method: Finding a Least Cost Solution

- There is one very important rule
- ***The number of occupied routes (or squares) must always be equal to one less than the sum of the number of rows plus the number of columns***
- In the Executive Furniture problem this means the initial solution must have 3 + 3 – 1 = 5 squares used

$$\text{Occupied shipping routes (squares)} = \text{Number of rows} + \text{Number of columns} - 1$$

- When the number of occupied rows is less than this, the solution is called ***degenerate***

Testing the Solution for Possible Improvement

- The stepping-stone method works by testing each unused square in the transportation table to see what would happen to total shipping costs if one unit of the product were tentatively shipped on an unused route
- There are five steps in the process

Five Steps to Test Unused Squares with the Stepping-Stone Method

1. Select an unused square to evaluate
2. Beginning at this square, trace a closed path back to the original square via squares that are currently being used with only horizontal or vertical moves allowed
3. Beginning with a plus (+) sign at the unused square, place alternate minus (–) signs and plus signs on each corner square of the closed path just traced

Five Steps to Test Unused Squares with the Stepping-Stone Method

4. Calculate an ***improvement index*** by adding together the unit cost figures found in each square containing a plus sign and then subtracting the unit costs in each square containing a minus sign
5. Repeat steps 1 to 4 until an improvement index has been calculated for all unused squares. If all indices computed are greater than or equal to zero, an optimal solution has been reached. If not, it is possible to improve the current solution and decrease total shipping costs.

Five Steps to Test Unused Squares with the Stepping-Stone Method

- For the Executive Furniture Corporation data

Steps 1 and 2. Beginning with Des Moines–Boston route we trace a closed path using only currently occupied squares, alternately placing plus and minus signs in the corners of the path

- In a ***closed path***, only squares currently used for shipping can be used in turning corners
- ***Only one*** closed route is possible for each square we wish to test

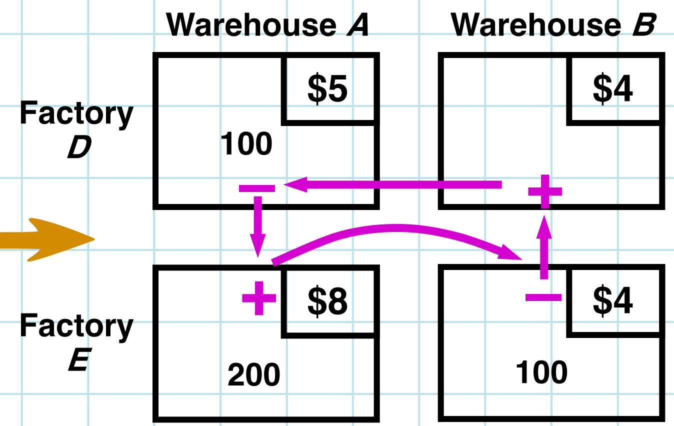
Five Steps to Test Unused Squares with the Stepping-Stone Method

Step 3. We want to test the cost-effectiveness of the Des Moines–Boston shipping route so we pretend we are shipping one desk from Des Moines to Boston and put a plus in that box

- But if we ship one ***more*** unit out of Des Moines we will be sending out 101 units
- Since the Des Moines factory capacity is only 100, we must ship ***fewer*** desks from Des Moines to Albuquerque so we place a minus sign in that box
- But that leaves Albuquerque one unit short so we must increase the shipment from Evansville to Albuquerque by one unit and so on until we complete the entire closed path

Five Steps to Test Unused Squares with the Stepping-Stone Method

- **Evaluating the unused Des Moines–Boston shipping route**

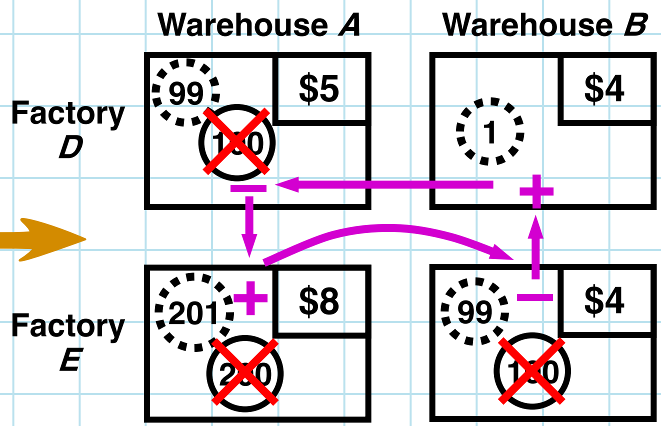


TO / FROM	ALBUQUERQUE	BOSTON	CLEVELAND	FACTORY CAPACITY
DES MOINES	100 ($5)	($4)	($3)	100
EVANSVILLE	200 ($8)	100 ($4)	($3)	300
FORT LAUDERDALE	($9)	100 ($7)	200 ($5)	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 10.4

Five Steps to Test Unused Squares with the Stepping-Stone Method

- **Evaluating the unused Des Moines–Boston shipping route**

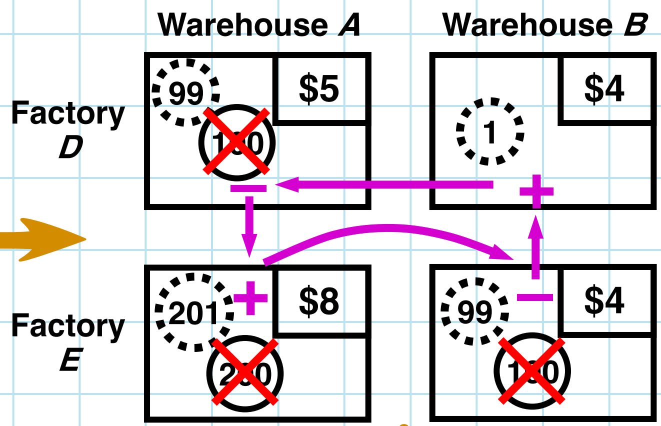


TO / FROM	ALBUQUERQUE	BOSTON	CLEVELAND	FACTORY CAPACITY
DES MOINES	100 ($5)	($4)	($3)	100
EVANSVILLE	200 ($8)	100 ($4)	($3)	300
FORT LAUDERDALE	($9)	100 ($7)	200 ($5)	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 10.4

Five Steps to Test Unused Squares with the Stepping-Stone Method

- **Evaluating the unused Des Moines–Boston shipping route**



FROM \ TO	ALBUQUERQUE	BOSTON	CLEVELAND	
DES MOINES	100 ($5)	($4)	($3)	
EVANSVILLE	200 ($8)	100 ($4)	($3)	
FORT LAUDERDALE	($9)	100 ($7)	200 ($5)	
WAREHOUSE REQUIREMENTS	300	200	200	700

Result of Proposed Shift in Allocation

= 1 x $4
– 1 x $5
+ 1 x $8
– 1 x $4 = +$3

Table 10.4

Five Steps to Test Unused Squares with the Stepping-Stone Method

Step 4. We can now compute an ***improvement index*** (I_{ij}) for the Des Moines–Boston route

- We add the costs in the squares with plus signs and subtract the costs in the squares with minus signs

Des Moines–Boston index $= I_{DB} = +\$4 - \$5 + \$5 - \$4 = +\$3$

- This means for every desk shipped via the Des Moines–Boston route, total transportation cost will ***increase*** by $3 over their current level

Five Steps to Test Unused Squares with the Stepping-Stone Method

Step 5. We can now examine the Des Moines–Cleveland unused route which is slightly more difficult to draw

- Again we can only turn corners at squares that represent existing routes
- We must pass through the Evansville–Cleveland square but we can not turn there or put a + or – sign
- The closed path we will use is

$$+ DC - DA + EA - EB + FB - FC$$

Five Steps to Test Unused Squares with the Stepping-Stone Method

- Evaluating the Des Moines–Cleveland shipping route

FROM \ TO	ALBUQUERQUE	BOSTON	CLEVELAND	FACTORY CAPACITY
DES MOINES	100 ($5) −	($4)	Start ($3) +	100
EVANSVILLE	200 ($8) +	100 ($4) −	($3)	300
FORT LAUDERDALE	($9)	100 ($7) +	200 ($5) −	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 10.5

Des Moines–Cleveland improvement index $= I_{DC} = +\$3 - \$5 + \$8 - \$4 + \$7 - \$5 = +\$4$

Five Steps to Test Unused Squares with the Stepping-Stone Method

- **Opening the Des Moines–Cleveland route will not lower our total shipping costs**
- **Evaluating the other two routes we find**

Evansville-Cleveland index $= I_{EC} = + \$3 - \$4 + \$7 - \$5 = + \$1$

- **The closed path is**

$$+ EC - EB + FB - FC$$

Fort Lauderdale–Albuquerque index $= I_{FA} = + \$9 - \$7 + \$4 - \$8 = - \$2$

- **The closed path is**

$$+ FA - FB + EB - EA$$

- **So opening the Fort Lauderdale-Albuquerque route *will* lower our total transportation costs**

Obtaining an Improved Solution

- **In the Executive Furniture problem there is only one unused route with a negative index (Fort Lauderdale-Albuquerque)**
- **If there was more than one route with a negative index, we would choose the one with the largest improvement**
- **We now want to ship the maximum allowable number of units on the new route**
- **The quantity to ship is found by referring to the closed path of plus and minus signs for the new route and selecting the *smallest number* found in those squares containing minus signs**

Obtaining an Improved Solution

- To obtain a new solution, that number is added to all squares on the closed path with plus signs and subtracted from all squares the closed path with minus signs
- All other squares are unchanged
- In this case, the maximum number that can be shipped is 100 desks as this is the smallest value in a box with a negative sign (*FB* route)
- We add 100 units to the *FA* and *EB* routes and subtract 100 from *FB* and *EA* routes
- This leaves balanced rows and columns and an improved solution

Obtaining an Improved Solution

- Stepping-stone path used to evaluate route *FA*

FROM \ TO	*A*	*B*	*C*	FACTORY CAPACITY
D	100 $5	$4	$3	100
E	200 $8 −	100 $4 +	$3	300
F	+ $9	− 100 $7	200 $5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 10.6

Obtaining an Improved Solution

- Second solution to the Executive Furniture problem

FROM \ TO	A	B	C	FACTORY CAPACITY
D	100 $5	$4	$3	100
E	100 $8	200 $4	$3	300
F	100 $9	$7	200 $5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 10.7

- Total shipping costs have been reduced by (100 units) x ($2 saved per unit) and now equals $4,000

Obtaining an Improved Solution

- This second solution may or may not be optimal
- To determine whether further improvement is possible, we return to the first five steps to test each square that is *now* unused
- The four new improvement indices are

D to *B* = I_{DB} = + $4 – $5 + $8 – $4 = + $3
(closed path: + *DB* – *DA* + *EA* – *EB*)

D to *C* = I_{DC} = + $3 – $5 + $9 – $5 = + $2
(closed path: + *DC* – *DA* + *FA* – *FC*)

E to *C* = I_{EC} = + $3 – $8 + $9 – $5 = – $1
(closed path: + *EC* – *EA* + *FA* – *FC*)

F to *B* = I_{FB} = + $7 – $4 + $8 – $9 = + $2
(closed path: + *FB* – *EB* + *EA* – *FA*)

Obtaining an Improved Solution

- Path to evaluate for the *EC* route

FROM \ TO	*A*	*B*	*C*	FACTORY CAPACITY
D	100 ($5)	($4)	($3)	100
E	100 ($8) −	200 ($4)	Start ($3) +	300
F	100 ($9) +	($7)	200 ($5) −	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 10.8

- An improvement can be made by shipping the maximum allowable number of units from *E* to *C*

Obtaining an Improved Solution

- Total cost of third solution

ROUTE		DESKS SHIPPED	x	PER UNIT COST ($)	=	TOTAL COST ($)
FROM	TO					
D	*A*	100		5		500
E	*B*	200		4		800
E	*C*	100		3		300
F	*A*	200		9		1,800
F	*C*	100		5		500
						3,900

Obtaining an Improved Solution

- **Third and optimal solution**

FROM \ TO	*A*	*B*	*C*	FACTORY CAPACITY
D	100 \$5	\$4	\$3	100
E	\$8	200 \$4	100 \$3	300
F	200 \$9	\$7	100 \$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 10.9

Obtaining an Improved Solution

- This solution is optimal as the improvement indices that can be computed are all greater than or equal to zero

D to *B* = I_{DB} = + \$4 – \$5 + \$9 – \$5 + \$3 – \$4 = **+ \$2**
(closed path: + *DB* – *DA* + *FA* – *FC* + *EC* – *EB*)

D to *C* = I_{DC} = + \$3 – \$5 + \$9 – \$5 = **+ \$2**
(closed path: + *DC* – *DA* + *FA* – *FC*)

E to *A* = I_{EA} = + \$8 – \$9 + \$5 – \$3 = **+ \$1**
(closed path: + *EA* – *FA* + *FC* – *EC*)

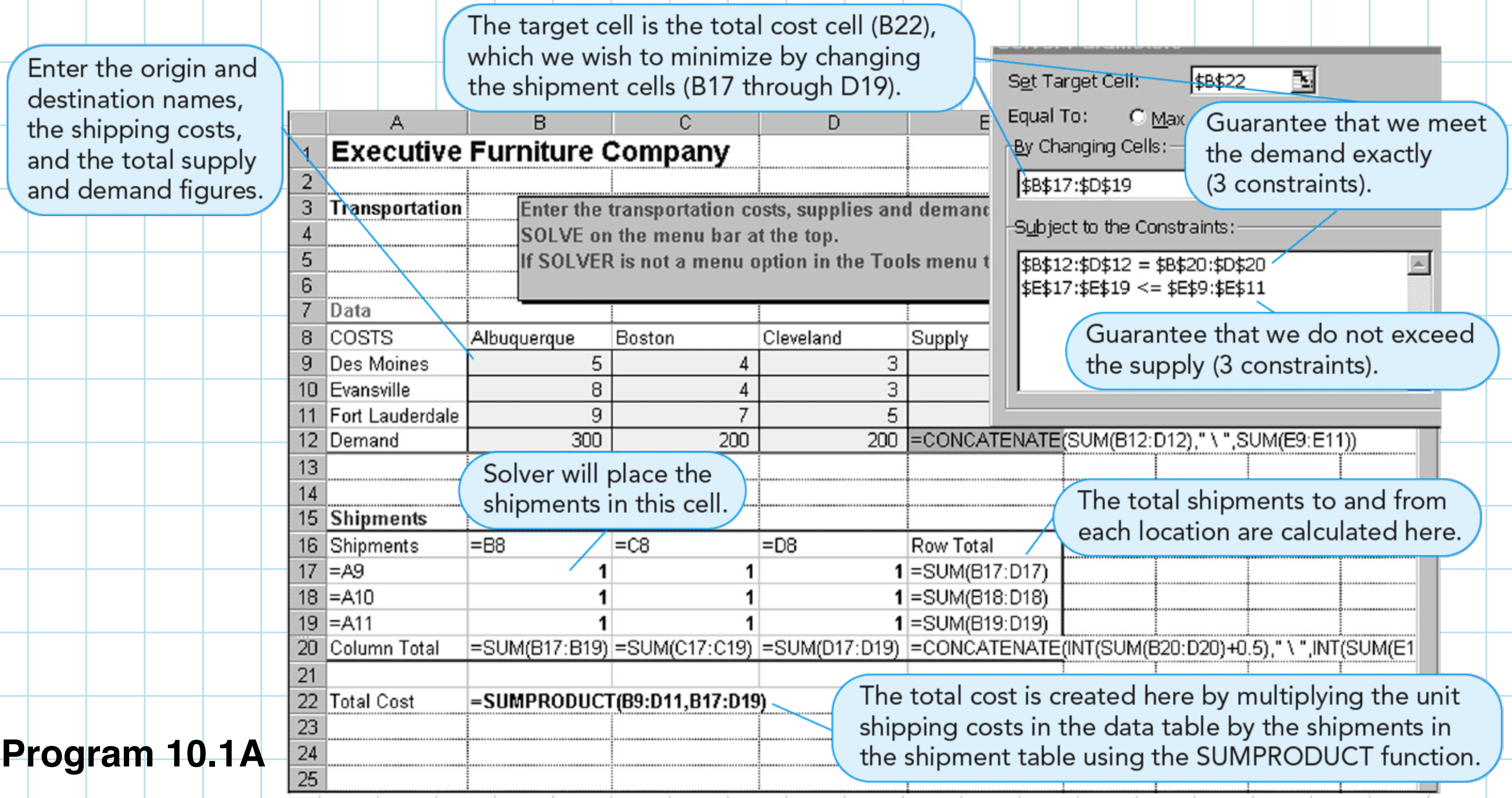
F to *B* = I_{FB} = + \$7 – \$5 + \$3 – \$4 = **+ \$1**
(closed path: + *FB* – *FC* + *EC* – *EB*)

Summary of Steps in Transportation Algorithm (Minimization)

1. Set up a balanced transportation table
2. Develop initial solution using either the northwest corner method or Vogel's approximation method
3. Calculate an improvement index for each empty cell using either the stepping-stone method or the MODI method. If improvement indices are all nonnegative, stop as the optimal solution has been found. If any index is negative, continue to step 4.
4. Select the cell with the improvement index indicating the greatest decrease in cost. Fill this cell using the stepping-stone path and go to step 3.

Using Excel QM to Solve Transportation Problems

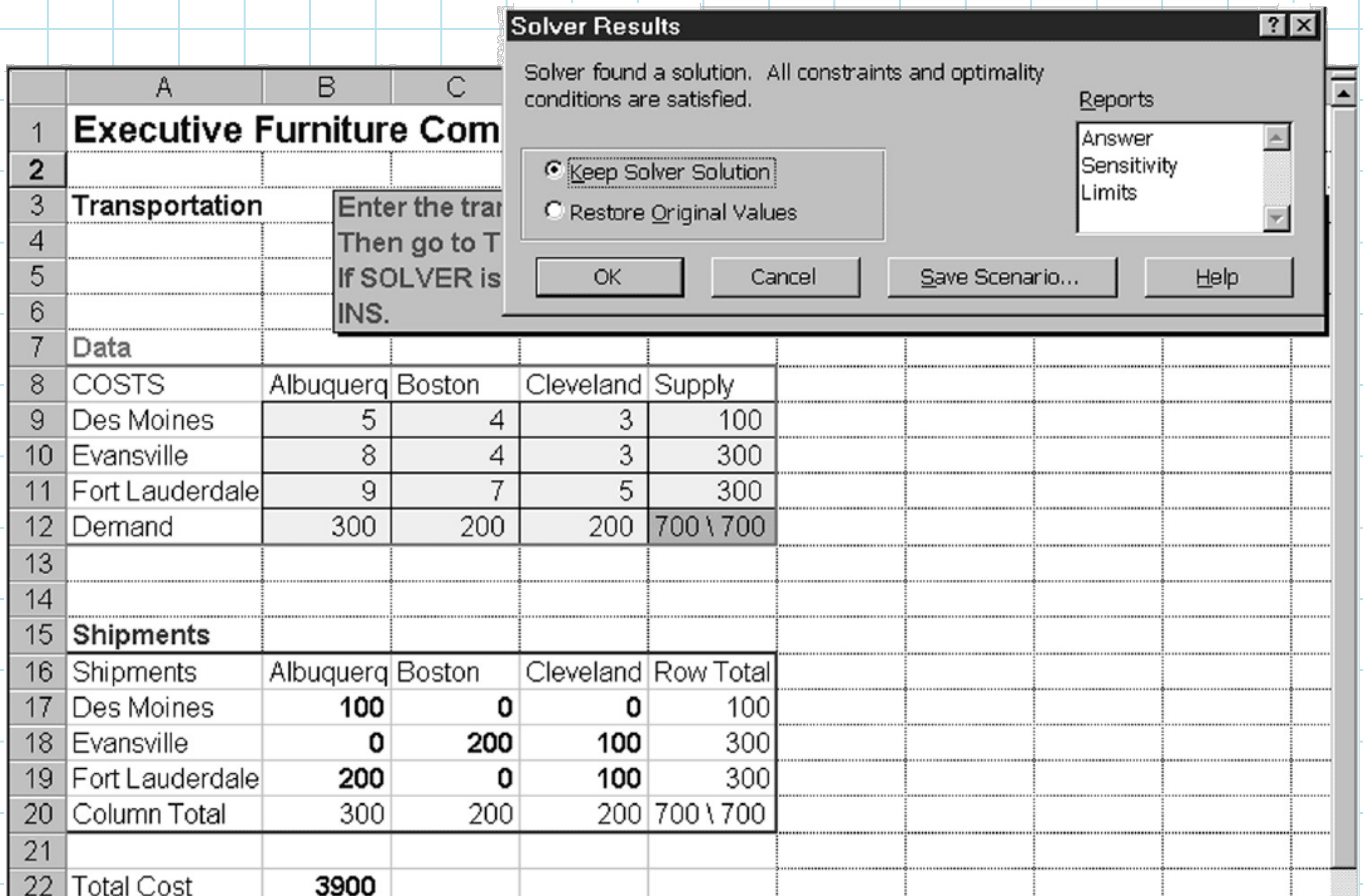
- **Excel QM input screen and formulas**



Program 10.1A

Using Excel QM to Solve Transportation Problems

- **Output from Excel QM with optimal solution**



	A	B	C	D	E
1	**Executive Furniture Com**				
2					
3	**Transportation**				
4					
5					
6					
7	Data				
8	COSTS	Albuquerq	Boston	Cleveland	Supply
9	Des Moines	5	4	3	100
10	Evansville	8	4	3	300
11	Fort Lauderdale	9	7	5	300
12	Demand	300	200	200	700 \ 700
13					
14					
15	**Shipments**				
16	Shipments	Albuquerq	Boston	Cleveland	Row Total
17	Des Moines	**100**	**0**	**0**	100
18	Evansville	**0**	**200**	**100**	300
19	Fort Lauderdale	**200**	**0**	**100**	300
20	Column Total	300	200	200	700 \ 700
21					
22	Total Cost	**3900**			

Program 10.1B

MODI Method

- The MODI (*modified distribution*) method allows us to compute improvement indices quickly for each unused square without drawing all of the closed paths
- Because of this, it can often provide considerable time savings over the stepping-stone method for solving transportation problems
- If there is a negative improvement index, then only one stepping-stone path must be found
- This is used in the same manner as before to obtain an improved solution

How to Use the MODI Approach

- In applying the MODI method, we begin with an initial solution obtained by using the northwest corner rule
- We now compute a value for each row (call the values R_1, R_2, R_3 if there are three rows) and for each column (K_1, K_2, K_3) in the transportation table
- In general we let

  R_i = value for assigned row i

  K_j = value for assigned column j

  C_{ij} = cost in square ij (cost of shipping from source i to destination j)

Five Steps in the MODI Method to Test Unused Squares

1. Compute the values for each row and column, set

   $$R_i + K_j = C_{ij}$$

   but only for those squares that are currently used or occupied
2. After all equations have been written, set $R_1 = 0$
3. Solve the system of equations for R and K values
4. Compute the improvement index for each unused square by the formula

   $$\text{Improvement Index } (I_{ij}) = C_{ij} - R_i - K_j$$
5. Select the best negative index and proceed to solve the problem as you did using the stepping-stone method

Solving the Executive Furniture Corporation Problem with MODI

- The initial northwest corner solution is repeated in Table 10.10
- Note that to use the MODI method we have added the R_is (rows) and K_js (columns)

	K_j	K_1	K_2	K_3	
R_i	FROM \ TO	*A*	*B*	*C*	FACTORY CAPACITY
R_1	*D*	100 \$5	\$4	\$3	100
R_2	*E*	200 \$8	100 \$4	\$3	300
R_3	*F*	\$9	100 \$7	200 \$5	300
	WAREHOUSE REQUIREMENTS	300	200	200	700

Table 10.10

Solving the Executive Furniture Corporation Problem with MODI

- The first step is to set up an equation for each occupied square
- By setting $R_1 = 0$ we can easily solve for K_1, R_2, K_2, R_3, and K_3

(1)	$R_1 + K_1 = 5$	$0 + K_1 = 5$	$K_1 = 5$
(2)	$R_2 + K_1 = 8$	$R_2 + 5 = 8$	$R_2 = 3$
(3)	$R_2 + K_2 = 4$	$3 + K_2 = 4$	$K_2 = 1$
(4)	$R_3 + K_2 = 7$	$R_3 + 1 = 7$	$R_3 = 6$
(5)	$R_3 + K_3 = 5$	$6 + K_3 = 5$	$K_3 = -1$

Solving the Executive Furniture Corporation Problem with MODI

- The next step is to compute the improvement index for each unused cell using the formula

  Improvement index (I_{ij}) = $C_{ij} - R_i - K_j$

- We have

Des Moines-Boston index	I_{DB}	$= C_{12} - R_1 - K_2 = 4 - 0 - 1$ $= +\$3$
Des Moines-Cleveland index	I_{DC}	$= C_{13} - R_1 - K_3 = 3 - 0 - (-1)$ $= +\$4$
Evansville-Cleveland index	I_{EC}	$= C_{23} - R_2 - K_3 = 3 - 3 - (-1)$ $= +\$1$
Fort Lauderdale-Albuquerque index	I_{FA}	$= C_{31} - R_3 - K_1 = 9 - 6 - 5$ $= -\$2$

Solving the Executive Furniture Corporation Problem with MODI

- **The steps we follow to develop an improved solution after the improvement indices have been computed are**
  1. **Beginning at the square with the best improvement index, trace a closed path back to the original square via squares that are currently being used**
  2. **Beginning with a plus sign at the unused square, place alternate minus signs and plus signs on each corner square of the closed path just traced**

Solving the Executive Furniture Corporation Problem with MODI

3. Select the smallest quantity found in those squares containing the minus signs and add that number to all squares on the closed path with plus signs; subtract the number from squares with minus signs
4. Compute new improvement indices for this new solution using the MODI method
   - Note that new R_i and K_j values must be calculated

- Follow this procedure for the second and third solutions

Vogel's Approximation Method: Another Way To Find An Initial Solution

- *Vogel's Approximation Method* (*VAM*) is not as simple as the northwest corner method, but it provides a very good initial solution, often one that is the *optimal* solution
- VAM tackles the problem of finding a good initial solution by taking into account the costs associated with each route alternative
- This is something that the northwest corner rule does not do
- To apply VAM, we first compute for each row and column the penalty faced if we should ship over the *second-best* route instead of the *least-cost* route

Vogel's Approximation Method

- The six steps involved in determining an initial VAM solution are illustrated below beginning with the same layout originally shown in Table 10.2

VAM Step 1. For each row and column of the transportation table, find the difference between the distribution cost on the *best* route in the row or column and the *second best* route in the row or column

- This is the *opportunity cost* of not using the best route
- Step 1 has been done in Table 10.11

Vogel's Approximation Method

- Transportation table with VAM row and column differences shown

	3	0	0 ← OPPORTUNITY COSTS		
FROM \ TO	*A*	*B*	*C*	TOTAL AVAILABLE	↓
D	100 \$5	\$4	\$3	100	1
E	200 \$8	100 \$4	\$3	300	1
F	\$9	100 \$7	200 \$5	300	2
TOTAL REQUIRED	300	200	200	700	

Table 10.11

Vogel's Approximation Method

VAM Step 2. identify the row or column with the greatest opportunity cost, or difference (column *A* in this example)

VAM Step 3.Assign as many units as possible to the lowest-cost square in the row or column selected

VAM Step 4. Eliminate any row or column that has been completely satisfied by the assignment just made by placing *X*s in each appropriate square

VAM Step 5. Recompute the cost differences for the transportation table, omitting rows or columns eliminated in the previous step

Vogel's Approximation Method

- **VAM assignment with *D*'s requirements satisfied**

	~~3~~ 1	~~0~~ 3	~~0~~ 2 ← OPPORTUNITY COSTS		
FROM \ TO	*A*	*B*	*C*	TOTAL AVAILABLE	
D	100 $5	X $4	X $3	100	1
E	$8	$4	$3	300	1
F	$9	$7	$5	300	2
TOTAL REQUIRED	300	200	200	700	

Table 10.12

Vogel's Approximation Method

VAM Step 6. Return to step 2 for the rows and columns remaining and repeat the steps until an initial feasible solution has been obtained

- In this case column *B* now has the greatest difference, 3
- We assign 200 units to the lowest-cost square in the column, *EB*
- We recompute the differences and find the greatest difference is now in row *E*
- We assign 100 units to the lowest-cost square in the column, *EC*

Vogel's Approximation Method

- Second VAM assignment with *B*'s requirements satisfied

	~~3~~ 1	~~0~~ 3	~~0~~ 2	← OPPORTUNITY COSTS	
FROM \ TO	*A*	*B*	*C*	TOTAL AVAILABLE	↓
D	100 ($5)	X ($4)	X ($3)	100	1
E	($8)	200 ($4)	($3)	300	1
F	($9)	X ($7)	($5)	300	2
TOTAL REQUIRED	300	200	200	700	

Table 10.13

Vogel's Approximation Method

- Third VAM assignment with *E*'s requirements satisfied

FROM \ TO	*A*	*B*	*C*	TOTAL AVAILABLE
D	100 $5	X $4	X $3	100
E	X $8	200 $4	100 $3	300
F	$9	X $7	$5	300
TOTAL REQUIRED	300	200	200	700

Table 10.14

Vogel's Approximation Method

- Final assignments to balance column and row requirements

FROM \ TO	A	B	C	TOTAL AVAILABLE
D	100 ($5)	X ($4)	X ($3)	100
E	X ($8)	200 ($4)	100 ($3)	300
F	200 ($9)	X ($7)	100 ($5)	300
TOTAL REQUIRED	300	200	200	700

Table 10.15

Unbalanced Transportation Problems

- In real-life problems, total demand is frequently not equal to total supply
- These ***unbalanced problems*** can be handled easily by introducing ***dummy sources*** or ***dummy destinations***
- If total supply is greater than total demand, a dummy destination (warehouse), with demand exactly equal to the surplus, is created
- If total demand is greater than total supply, we introduce a dummy source (factory) with a supply equal to the excess of demand over supply

Unbalanced Transportation Problems

- In either case, shipping cost coefficients of zero are assigned to each dummy location or route as no goods will actually be shipped
- Any units assigned to a dummy destination represent excess capacity
- Any units assigned to a dummy source represent unmet demand

Demand Less Than Supply

- Suppose that the Des Moines factory increases its rate of production from 100 to 250 desks
- The firm is now able to supply a total of 850 desks each period
- Warehouse requirements remain the same (700) so the row and column totals do not balance
- We add a dummy column that will represent a fake warehouse requiring 150 desks
- This is somewhat analogous to adding a slack variable
- We use the northwest corner rule and either stepping-stone or MODI to find the optimal solution

Demand Less Than Supply

- Initial solution to an unbalanced problem where demand is less than supply

FROM \ TO	A	B	C	DUMMY WAREHOUSE	TOTAL AVAILABLE
D	250 ($5)	($4)	($3)	(0)	250
E	50 ($8)	200 ($4)	50 ($3)	(0)	300
F	($9)	($7)	150 ($5)	150 (0)	300
WAREHOUSE REQUIREMENTS	300	200	200	150	850

Total cost = 250($5) + 50($8) + 200($4) + 50($3) + 150($5) + 150(0) = $3,350

Table 10.16

New Des Moines capacity

Demand Greater than Supply

- The second type of unbalanced condition occurs when total demand is greater than total supply
- In this case we need to add a dummy row representing a fake factory
- The new factory will have a supply exactly equal to the difference between total demand and total real supply
- The shipping costs from the dummy factory to each destination will be zero

Demand Greater than Supply

- **Unbalanced transportation table for Happy Sound Stereo Company**

FROM \ TO	WAREHOUSE *A*	WAREHOUSE *B*	WAREHOUSE *C*	PLANT SUPPLY
PLANT *W*	$6	$4	$9	200
PLANT *X*	$10	$5	$8	175
PLANT *Y*	$12	$7	$6	75
WAREHOUSE DEMAND	250	100	150	500 \ 450

Totals do not balance

Table 10.17

Demand Greater than Supply

- Initial solution to an unbalanced problem in which demand is greater than supply

FROM \ TO	WAREHOUSE *A*	WAREHOUSE *B*	WAREHOUSE *C*	PLANT SUPPLY
PLANT *W*	200 [$6]	[$4]	[$9]	200
PLANT *X*	50 [$10]	100 [$5]	25 [$8]	175
PLANT *Y*	[$12]	[$7]	75 [$6]	75
PLANT *Y*	[0]	[0]	50 [0]	50
WAREHOUSE DEMAND	250	100	150	500

Total cost of initial solution = 200($6) + 50($10) + 100($5) + 25($8) + 75($6) + $50(0) = $2,850

Table 10.18

Degeneracy in Transportation Problems

- *Degeneracy* occurs when the number of occupied squares or routes in a transportation table solution is less than the number of rows plus the number of columns minus 1
- Such a situation may arise in the initial solution or in any subsequent solution
- Degeneracy requires a special procedure to correct the problem since there are not enough occupied squares to trace a closed path for each unused route and it would be impossible to apply the stepping-stone method or to calculate the *R* and *K* values needed for the MODI technique

Degeneracy in Transportation Problems

- To handle degenerate problems, create an artificially occupied cell
- That is, place a zero (representing a fake shipment) in one of the unused squares and then treat that square as if it were occupied
- The square chosen must be in such a position as to allow all stepping-stone paths to be closed
- There is usually a good deal of flexibility in selecting the unused square that will receive the zero

Degeneracy in an Initial Solution

- The Martin Shipping Company example illustrates degeneracy in an initial solution
- They have three warehouses which supply three major retail customers
- Applying the northwest corner rule the initial solution has only four occupied squares
- This is less than the amount required to use either the stepping-stone or MODI method to improve the solution (3 rows + 3 columns – 1 = 5)
- To correct this problem, place a zero in an unused square, typically one adjacent to the last filled cell

Degeneracy in an Initial Solution

- **Initial solution of a degenerate problem**

FROM \ TO	CUSTOMER 1	CUSTOMER 2	CUSTOMER 3	WAREHOUSE SUPPLY
WAREHOUSE 1	$8 100	$2 0	$6	100
WAREHOUSE 2	$10 0	$9 100	$9 20	120
WAREHOUSE 3	$7	$10	$7 80	80
CUSTOMER DEMAND	100	100	100	300

Table 10.19

Possible choices of cells to address the degenerate solution

Degeneracy During Later Solution Stages

- A transportation problem can become degenerate after the initial solution stage if the filling of an empty square results in two or more cells becoming empty simultaneously
- This problem can occur when two or more cells with minus signs tie for the lowest quantity
- To correct this problem, place a zero in one of the previously filled cells so that only one cell becomes empty

Degeneracy During Later Solution Stages

- **Bagwell Paint Example**
  - **After one iteration, the cost analysis at Bagwell Paint produced a transportation table that was not degenerate but was not optimal**
  - **The improvement indices are**

factory *A* – warehouse 2 index = +2
factory *A* – warehouse 3 index = +1
factory *B* – warehouse 3 index = –15
factory *C* – warehouse 2 index = +11

Only route with a negative index

Degeneracy During Later Solution Stages

- Bagwell Paint transportation table

FROM \ TO	WAREHOUSE 1	WAREHOUSE 2	WAREHOUSE 3	FACTORY CAPACITY
FACTORY *A*	70 ($8)	($5)	($16)	70
FACTORY *B*	50 ($15)	80 ($10)	($7)	130
FACTORY *C*	30 ($3)	($9)	50 ($10)	80
WAREHOUSE REQUIREMENT	150	80	50	280

Table 10.20

Degeneracy During Later Solution Stages

- Tracing a closed path for the factory B – warehouse 3 route

FROM \ TO	WAREHOUSE 1	WAREHOUSE 3
FACTORY *B*	50 ($15)	($7)
FACTORY *C*	30 ($3)	50 ($10)

Table 10.21

- This would cause two cells to drop to zero
- We need to place an artificial zero in one of these cells to avoid degeneracy

More Than One Optimal Solution

- It is possible for a transportation problem to have multiple optimal solutions
- This happens when one or more of the improvement indices zero in the optimal solution
- This means that it is possible to design alternative shipping routes with the same total shipping cost
- The alternate optimal solution can be found by shipping the most to this unused square using a stepping-stone path
- In the real world, alternate optimal solutions provide management with greater flexibility in selecting and using resources

Maximization Transportation Problems

- If the objective in a transportation problem is to maximize profit, a minor change is required in the transportation algorithm
- Now the optimal solution is reached when all the improvement indices are negative or zero
- The cell with the largest positive improvement index is selected to be filled using a stepping-stone path
- This new solution is evaluated and the process continues until there are no positive improvement indices

Unacceptable Or Prohibited Routes

- At times there are transportation problems in which one of the sources is unable to ship to one or more of the destinations
- When this occurs, the problem is said to have an ***unacceptable*** or ***prohibited route***
- In a minimization problem, such a prohibited route is assigned a very high cost to prevent this route from ever being used in the optimal solution
- In a maximization problem, the very high cost used in minimization problems is given a negative sign, turning it into a very bad profit

Facility Location Analysis

- The transportation method is especially useful in helping a firm to decide where to locate a new factory or warehouse
- Each alternative location should be analyzed within the framework of one ***overall*** distribution system
- The new location that yields the minimum cost for the ***entire system*** is the one that should be chosen

Locating a New Factory for Hardgrave Machine Company

- Hardgrave Machine produces computer components at three plants and they ship to four warehouses
- The plants have not been able to keep up with demand so the firm wants to build a new plant
- Two sites are being considered, Seattle and Birmingham
- Data has been collected for each possible location
- Which new location will yield the lowest cost for the firm in combination with the existing plants and warehouses

Locating a New Factory for Hardgrave Machine Company

- Hardgrave's demand and supply data

WAREHOUSE	MONTHLY DEMAND (UNITS)	PRODUCTION PLANT	MONTHLY SUPPLY	COST TO PRODUCE ONE UNIT ($)
Detroit	10,000	Cincinnati	15,000	48
Dallas	12,000	Salt Lake	6,000	50
New York	15,000	Pittsburgh	14,000	52
Los Angeles	9,000		35,000	
	46,000			

Supply needed from new plant = 46,000 – 35,000 = 11,000 units per month

Table 10.22

ESTIMATED PRODUCTION COST PER UNIT AT PROPOSED PLANTS	
Seattle	$53
Birmingham	$49

Locating a New Factory for Hardgrave Machine Company

- **Hardgrave's shipping costs**

FROM \ TO	DETROIT	DALLAS	NEW YORK	LOS ANGELES
CINCINNATI	$25	$55	$40	$60
SALT LAKE	35	30	50	40
PITTSBURGH	36	45	26	66
SEATTLE	60	38	65	27
BIRMINGHAM	35	30	41	50

Table 10.23

Locating a New Factory for Hardgrave Machine Company

- **Optimal solution for the Birmingham location**

FROM \ TO	DETROIT	DALLAS	NEW YORK	LOS ANGELES	FACTORY CAPACITY
CINCINNATI	10,000 (73)	(103)	1,000 (88)	4,000 (108)	15,000
SALT LAKE	(85)	1,000 (80)	(100)	5,000 (90)	6,000
PITTSBURGH	(88)	(97)	14,000 (78)	(118)	14,000
BIRMINGHAM	(84)	11,000 (79)	(90)	(99)	11,000
WAREHOUSE REQUIREMENT	10,000	12,000	15,000	9,000	46,000

Table 10.24

Locating a New Factory for Hardgrave Machine Company

- **Optimal solution for the Seattle location**

FROM \ TO	DETROIT	DALLAS	NEW YORK	LOS ANGELES	FACTORY CAPACITY
CINCINNATI	10,000 (73)	4,000 (103)	1,000 (88)	(108)	15,000
SALT LAKE	(85)	6,000 (80)	(100)	(90)	6,000
PITTSBURGH	(88)	(97)	14,000 (78)	(118)	14,000
SEATTLE	(113)	2,000 (91)	(118)	9,000 (80)	11,000
WAREHOUSE REQUIREMENT	10,000	12,000	15,000	9,000	46,000

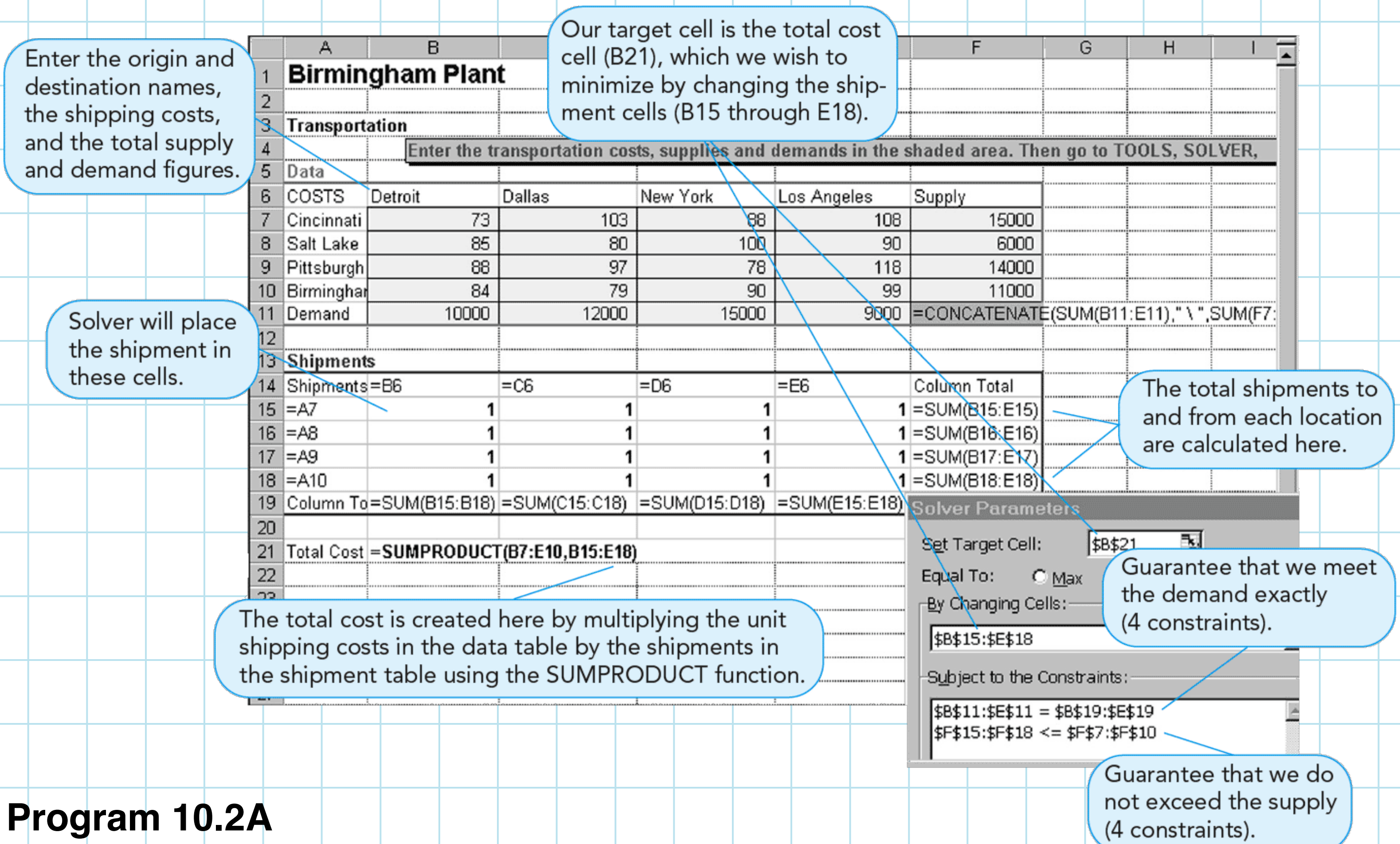
Table 10.25

Locating a New Factory for Hardgrave Machine Company

- By comparing the total system costs of the two alternatives, Hardgrave can select the lowest cost option
- The Birmingham location yields a total system cost of $3,741,000
- The Seattle location yields a total system cost of $3,704,000
- With the lower total system cost, the Seattle location is favored
- Excel QM can also be used as a solution tool

Locating a New Factory for Hardgrave Machine Company

- **Excel input screen**



Program 10.2A

Locating a New Factory for Hardgrave Machine Company

- **Output from Excel QM analysis**

Solver Results

Solver found a solution. All constraints and optimality conditions are satisfied.

Reports: Answer, Sensitivity, Limits

- Keep Solver Solution
- Restore Original Values

OK | Cancel | Save Scenario... | Help

	A	B	C	D	E	F
1	**Birmingham Plant**					
2						
3	**Transportation**					
4		Enter the transportati				
5	Data					
6	COSTS	Detroit	Dallas	New York	Los Angel	Supply
7	Cincinnati	73	103	88	108	15000
8	Salt Lake	85	80	100	90	6000
9	Pittsburgh	88	97	78	118	14000
10	Birmingha	84	79	90	99	11000
11	Demand	10000	12000	15000	9000	46000 \ 46000
12						
13	**Shipments**					
14	Shipments	Detroit	Dallas	New York	Los Angel	Column To
15	Cincinnati	**10000**	**0**	**1000**	**4000**	15000
16	Salt Lake	**0**	**1000**	**0**	**5000**	6000
17	Pittsburgh	**0**	**0**	**14000**	**0**	14000
18	Birmingha	**0**	**11000**	**0**	**0**	11000
19	Column To	10000	12000	15000	9000	46000 \ 46000
20						
21	Total Cost	**3741000**				

Program 10.2A

Assignment Model Approach

- The second special-purpose LP algorithm is the assignment method
- Each assignment problem has associated with it a table, or matrix
- Generally, the rows contain the objects or people we wish to assign, and the columns comprise the tasks or things we want them assigned to
- The numbers in the table are the costs associated with each particular assignment
- An assignment problem can be viewed as a transportation problem in which the capacity from each source is 1 and the demand at each destination is 1

Assignment Model Approach

- The Fix-It Shop has three rush projects to repair
- They have three repair persons with different talents and abilities
- The owner has estimates of wage costs for each worker for each project
- The owner's objective is to assign the three project to the workers in a way that will result in the lowest cost to the shop
- Each project will be assigned exclusively to one worker

Assignment Model Approach

- Estimated project repair costs for the Fix-It shop assignment problem

	PROJECT		
PERSON	1	2	3
Adams	$11	$14	$6
Brown	8	10	11
Cooper	9	12	7

Table 10.26

Assignment Model Approach

- Summary of Fix-It Shop assignment alternatives and costs

PRODUCT ASSIGNMENT				
1	2	3	LABOR COSTS ($)	TOTAL COSTS ($)
Adams	Brown	Cooper	11 + 10 + 7	28
Adams	Cooper	Brown	11 + 12 + 11	34
Brown	Adams	Cooper	8 + 14 + 7	29
Brown	Cooper	Adams	8 + 12 + 6	26
Cooper	Adams	Brown	9 + 14 + 11	34
Cooper	Brown	Adams	9 + 10 + 6	25

Table 10.27

The Hungarian Method (Flood's Technique)

- The ***Hungarian method*** is an efficient method of finding the optimal solution to an assignment problem without having to make direct comparisons of every option
- It operates on the principle of ***matrix reduction***
- By subtracting and adding appropriate numbers in the cost table or matrix, we can reduce the problem to a matrix of ***opportunity costs***
- Opportunity costs show the relative penalty associated with assigning any person to a project as opposed to making the ***best*** assignment
- We want to make assignment so that the opportunity cost for each assignment is zero

Three Steps of the Assignment Method

1. ***Find the opportunity cost table by***:
   (a) Subtracting the smallest number in each row of the original cost table or matrix from every number in that row
   (b) Then subtracting the smallest number in each column of the table obtained in part (a) from every number in that column
2. ***Test the table resulting from step 1 to see whether an optimal assignment can be made*** by drawing the minimum number of vertical and horizontal straight lines necessary to cover all the zeros in the table. If the number of lines is less than the number of rows or columns, proceed to step 3.

Three Steps of the Assignment Method

3. ***Revise the present opportunity cost table*** by subtracting the smallest number not covered by a line from every other uncovered number. This same number is also added to any number(s) lying at the intersection of horizontal and vertical lines. Return to step 2 and continue the cycle until an optimal assignment is possible.

Steps in the Assignment Method

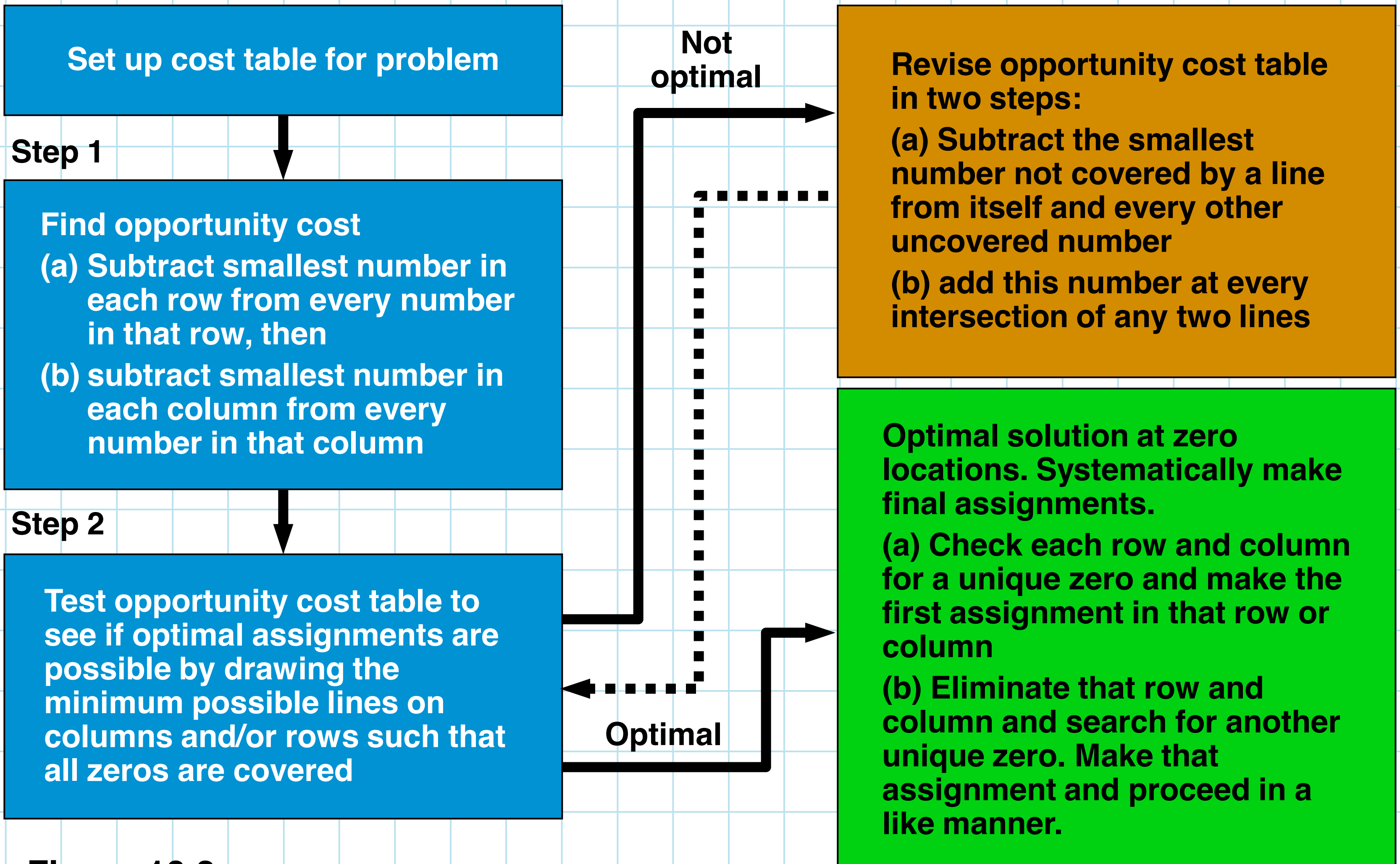


Figure 10.3

The Hungarian Method (Flood's Technique)

- Step 1: Find the opportunity cost table
  - We can compute ***row*** opportunity costs and ***column*** opportunity costs
  - What we need is the ***total*** opportunity cost
  - We derive this by taking the row opportunity costs and subtract the smallest number in that column from each number in that column

The Hungarian Method (Flood's Technique)

- **Cost of each person-project assignment**

PERSON	PROJECT 1	PROJECT 2	PROJECT 3
Adams	$11	$14	$6
Brown	8	10	11
Cooper	9	12	7

Table 10.28

- **Row opportunity cost table**

PERSON	PROJECT 1	PROJECT 2	PROJECT 3
Adams	$5	$8	$0
Brown	0	2	3
Cooper	2	5	0

Table 10.29

- **The opportunity cost of assigning Cooper to project 2 is $12 – $7 = $5**

The Hungarian Method (Flood's Technique)

- We derive the total opportunity costs by taking the costs in Table 29 and subtract the smallest number in each column from each number in that column

- Row opportunity cost table

	PROJECT		
PERSON	1	2	3
Adams	$5	$8	$0
Brown	0	2	3
Cooper	2	5	0

Table 10.29

- Total opportunity cost table

	PROJECT		
PERSON	1	2	3
Adams	$5	$6	$0
Brown	0	0	3
Cooper	2	3	0

Table 10.30

The Hungarian Method (Flood's Technique)

- Step 2: Test for the optimal assignment
  - We want to assign workers to projects in such a way that the total labor costs are at a minimum
  - We would like to have a total assigned opportunity cost of zero
  - The test to determine if we have reached an optimal solution is simple
  - We find the ***minimum*** number of straight lines necessary to cover all the zeros in the table
  - If the number of lines equals the number of rows or columns, an optimal solution has been reached

The Hungarian Method (Flood's Technique)

- Test for optimal solution

	PROJECT		
PERSON	1	2	3
Adams	$5	$6	$0
Brown	0	0	0
Cooper	2	3	0

Covering line 1

Covering line 2

Table 10.31

- This requires only two lines to cover the zeros so the solution is not optimal

The Hungarian Method (Flood's Technique)

- **Step 3: Revise the opportunity-cost table**
  - **We *subtract* the smallest number not covered by a line from all numbers not covered by a straight line**
  - **The same number is added to every number lying at the intersection of any two lines**
  - **We then return to step 2 to test this new table**

The Hungarian Method (Flood's Technique)

- **Revised opportunity cost table (derived by subtracting 2 from each cell not covered by a line and adding 2 to the cell at the intersection of the lines)**

	PROJECT		
PERSON	1	2	3
Adams	$3	$4	$0
Brown	0	0	5
Cooper	0	1	0

Table 10.32

The Hungarian Method (Flood's Technique)

- **Optimality test on the revised opportunity cost table**

	PROJECT		
PERSON	1	2	3
Adams	$3	$4	$0
Brown	0	0	5
Cooper	0	1	0

Covering line 2

Table 10.33 Covering line 1 Covering line 3

- **This requires three lines to cover the zeros so the solution is optimal**

Making the Final Assignment

- **The optimal assignment is Adams to project 3, Brown to project 2, and Cooper to project 1**
- **But this is a simple problem**
- **For larger problems one approach to making the final assignment is to select a row or column that contains only one zero**
- **Make the assignment to that cell and rule out its row and column**
- **Follow this same approach for all the remaining cells**

Making the Final Assignment

- Total labor costs of this assignment are

ASSIGNMENT	COST ($)
Adams to project 3	6
Brown to project 2	10
Cooper to project 1	9
Total cost	25

Making the Final Assignment

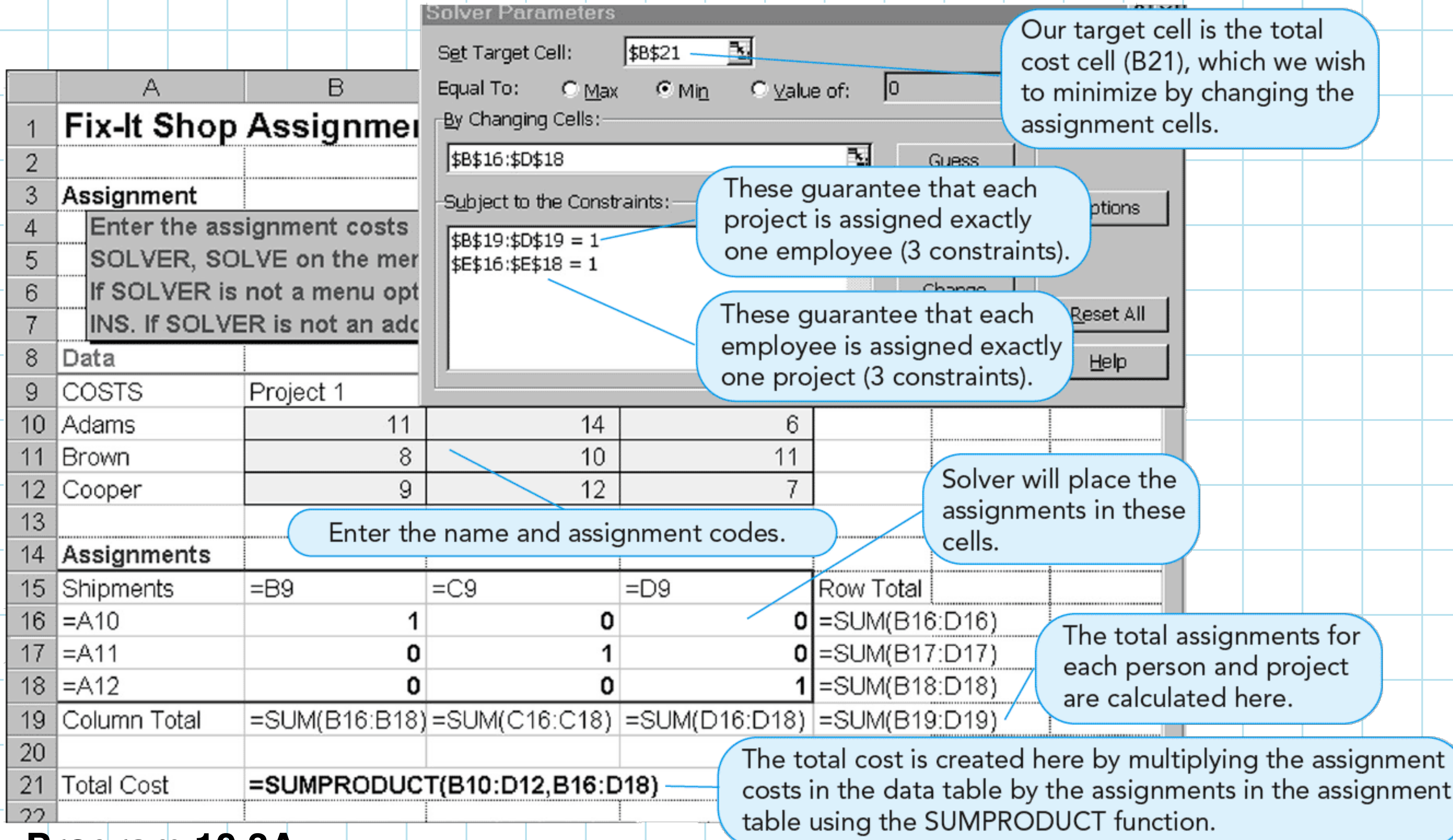
- Making the final assignments

(A) FIRST ASSIGNMENT				(B) SECOND ASSIGNMENT				(C) THIRD ASSIGNMENT			
	1	2	3		1	2	3		1	2	3
Adams	3	4	0	Adams	3	4	0	Adams	3	4	0
Brown	0	0	5	Brown	0	0	5	Brown	0	0	5
Cooper	0	1	0	Cooper	0	1	0	Cooper	0	1	0

Table 10.34

Using Excel QM for the Fix-It Shop Assignment Problem

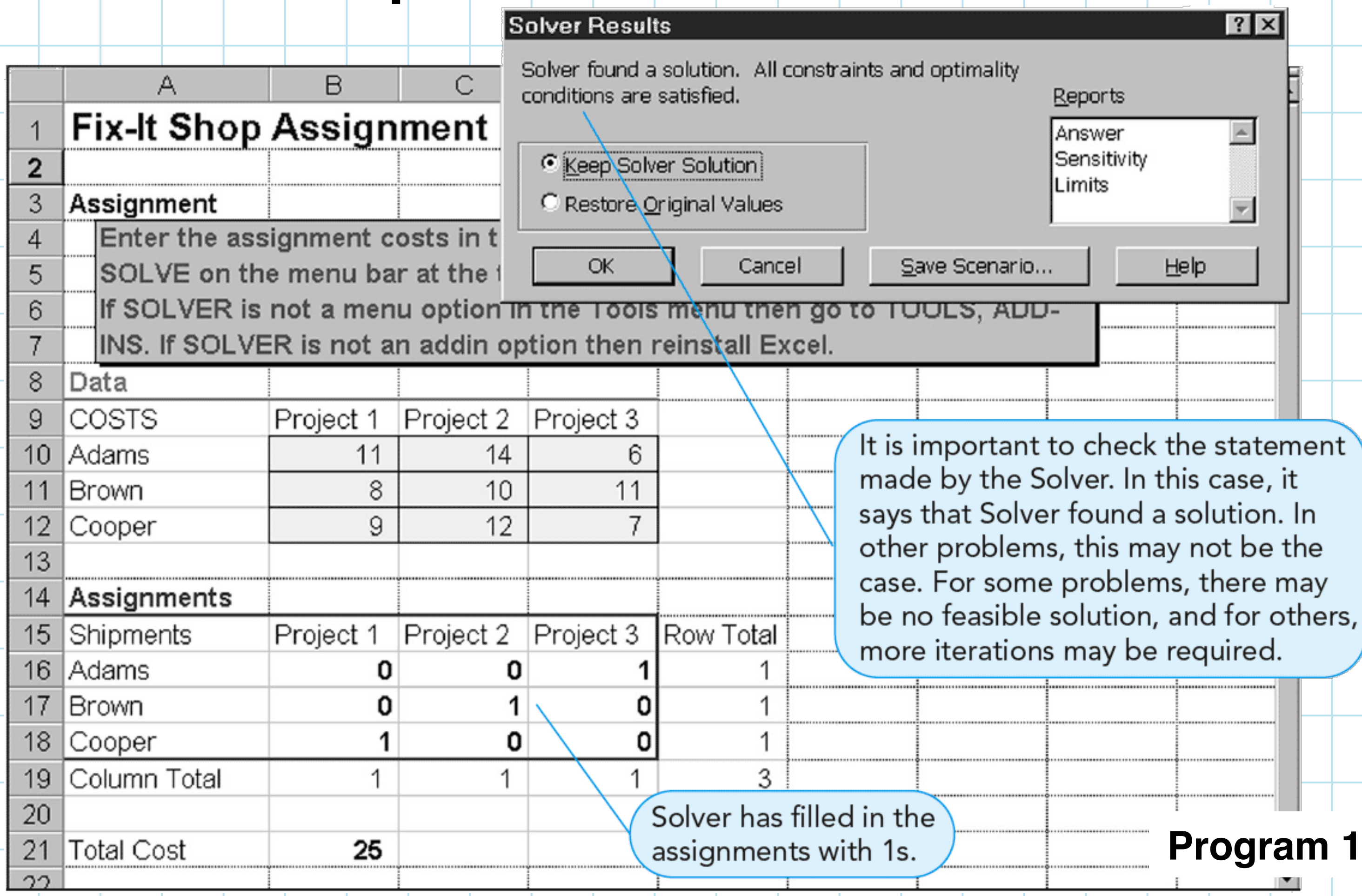
- **Excel QM assignment module**



Program 10.3A

Using Excel QM for the Fix-It Shop Assignment Problem

- **Excel QM output screen**



	A	B	C	D	E
1	**Fix-It Shop Assignment**				
2					
3	**Assignment**				
4	Enter the assignment costs in t				
5	SOLVE on the menu bar at the t				
6	If SOLVER is not a menu option in the Tools menu then go to TOOLS, ADD-				
7	INS. If SOLVER is not an addin option then reinstall Excel.				
8	Data				
9	COSTS	Project 1	Project 2	Project 3	
10	Adams	11	14	6	
11	Brown	8	10	11	
12	Cooper	9	12	7	
13					
14	**Assignments**				
15	Shipments	Project 1	Project 2	Project 3	Row Total
16	Adams	**0**	**0**	**1**	1
17	Brown	**0**	**1**	**0**	1
18	Cooper	**1**	**0**	**0**	1
19	Column Total	1	1	1	3
20					
21	Total Cost	**25**			

Program 10.3A

Unbalanced Assignment Problems

- Often the number of people or objects to be assigned does not equal the number of tasks or clients or machines listed in the columns, and the problem is ***unbalanced***
- When this occurs, and there are more rows than columns, simply add a ***dummy column*** or task
- If the number of tasks exceeds the number of people available, we add a ***dummy row***
- Since the dummy task or person is nonexistent, we enter zeros in its row or column as the cost or time estimate

Unbalanced Assignment Problems

- The Fix-It Shop has another worker available
- The shop owner still has the same basic problem of assigning workers to projects
- But the problem now needs a dummy column to balance the four workers and three projects

	PROJECT			
PERSON	1	2	3	DUMMY
Adams	$11	$14	$6	$0
Brown	8	10	11	0
Cooper	9	12	7	0
Davis	10	13	8	0

Table 10.35

Maximization Assignment Problems

- Some assignment problems are phrased in terms of maximizing the payoff, profit, or effectiveness
- It is easy to obtain an equivalent minimization problem by converting all numbers in the table to opportunity costs
- This is brought about by subtracting every number in the original payoff table from the largest single number in that table
- Transformed entries represent opportunity costs
- Once the optimal assignment has been found, the total payoff is found by adding the original payoffs of those cells that are in the optimal assignment

Maximization Assignment Problems

- The British navy wishes to assign four ships to patrol four sectors of the North Sea
- Ships are rated for their probable efficiency in each sector
- The commander wants to determine patrol assignments producing the greatest overall efficiencies

Maximization Assignment Problems

- Efficiencies of British ships in patrol sectors

	SECTOR			
SHIP	*A*	*B*	*C*	*D*
1	20	60	50	55
2	60	30	80	75
3	80	100	90	80
4	65	80	75	70

Table 10.36

Maximization Assignment Problems

- Opportunity cost of British ships

	SECTOR			
SHIP	*A*	*B*	*C*	*D*
1	80	40	50	45
2	40	70	20	25
3	20	0	10	20
4	35	20	25	30

Table 10.37

Maximization Assignment Problems

- **First convert the maximization efficiency table into a minimizing opportunity cost table by subtracting each rating from 100, the largest rating in the whole table**
- **The smallest number in each row is subtracted from every number in that row and the smallest number in each column is subtracted from every number in that column**
- **The minimum number of lines needed to cover the zeros in the table is four, so this represents an optimal solution**

Maximization Assignment Problems

- The overall efficiency

ASSIGNMENT	EFFICIENCY
Ship 1 to sector *D*	55
Ship 2 to sector *C*	80
Ship 3 to sector *B*	100
Ship 4 to sector *A*	65
Total efficiency	300